7 EQUATIONS FAMUS

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1. SOLUTION OF THE EQUATION $z \cdot e^{z} = t$

The roots of the equation play a role in the iteration of the exponential function[2;3;11] and in the solution and application of certain difference – Equation [1;9;10;12]. For this reason, several authors [4; 5; 7; 8; 9; 12] have found various properties of some or all of the roots. There is a work by **E. M. Write,** communicated by **Richard Bellman**, December 15, 1958. Also must mention a very important offer of Wolfram in Mathematica program with the W--Function.

But now we will solve the with the method (**G .R.E),** because it is the only method that throws ample light on general solve all equations. All the roots of our equation are given by $\log(z) + z = \log(t) + 2 \cdot k \cdot \pi \cdot i$ **(1)** where k takes all integral values as $k = 0, \pm 1, \pm 2, \pm 3, ... \pm \infty$. To solve the equation looking at three intervals, which in part are common and others differ in the method we choose.

A) Because we take the logarithm in both parties of the equation, the case $t < 0 \wedge t \in R$ leads only in complex roots. From the theory (G.R.E) we get two cases according to relation (I), because the relationship **(1)** has two functions, $P_1(z) = z = \zeta$ **(a)** and $P_2(z) = \log(z) = \zeta$ **(b)**.

Thus the first case **(a)** the solution we are the roots of the equation $$z_k = \zeta + \sum_{i=1}^{\infty} (\frac{(-m)^i}{Gamma\,(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (\zeta' \cdot \log^i(\zeta)))$$ **(i)** where ζ' is the first derivative of ζ with the type $\zeta = \log(t) + 2 \cdot k \cdot \pi \cdot i$ and k is integer , for a value of i .Also the case when t and is a complex number and especially when $|t| \geq e$, then the solution is represented by the same form **(i)**.

B) For interval $0 \leq t \leq \frac{1}{e} \wedge t \in R$ but also general where $0 \leq |t| \leq \frac{1}{e}$ in case that t is complex number and when $k \neq 0$, then the solutions illustrated from the equation :

$$z_k = \zeta + \sum_{i=1}^{\infty} (\frac{(-m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (\zeta' \cdot \log^i(\zeta)))$$ **(i)**

and in case that $k = 0$ then using the form $P_2(z) = \log(z) = \zeta \Rightarrow z = Exp(\zeta)$ the Lagrange equation from (G.R.E) transformed to $z_k = Exp(\zeta) + \sum_{i=1}^{\infty} (\frac{(-m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (Exp(\zeta)' \cdot Exp^i(\zeta)))$ **(ii)** but this specific

form translatable to $z_k = Exp(\zeta) + \sum_{i=1}^{\infty}(\frac{(-m)^i}{Gamma(i+1)}(i+1)^{i-1}\cdot Exp^{i+1}(\zeta))$ because we know the nth derivative of $Exp(m\cdot x) = m^n \cdot Exp(m\cdot x)$.

C) Specificity for the region $\frac{1}{e} \le t \le e \wedge t \in R$ but more generally $\frac{1}{e} \le |t| \le e$. Appears a small anomaly in the form (i) and as regards the complex or real value for k = 0 in $\zeta = \log(t) + 2\cdot k\cdot\pi\cdot i$. The case for Complex roots we get as a solution of the equation by the form

$$z_k = \zeta + \sum_{i=1}^{\infty}(\frac{(-m)^i}{Gamma(i+1)}\frac{\partial^{i-1}}{\partial\zeta^{i-1}}(\zeta'\cdot\log^i(\zeta)))\text{ except if } k \ne 0.$$

Eventually the case k = 0 is presented and the anomaly in the approach of the infinite sum in the form (ii)

$$z_s = Exp(\zeta) + \sum_{i=1}^{\infty}(\frac{(-m_*)^i}{Gamma(i+1)}(i+1)^{i-1}\cdot Exp^{i+1}(\zeta))\text{ but }\quad m_* = m/e^{s+1}\text{ with s>=1}$$

Because the replay will be s times and $\zeta = z_{s-1}, s>1$ we have to repeat. A very good approximation also in this special case is when we use the method approximate of Newton after obtaining an initial root z_s with s = 1 .

2. MAXIMUM THE SURFACE AREA AND VOLUME OF A HYPERSPHERE N DIMENSIONS

In mathematics, an n-sphere is a generalization of the surface of an ordinary sphere to arbitrary dimension. For any natural number n, an n-sphere of radius r is defined as the set of points in (n + 1)-dimensional Euclidean space which are at distance r from a central point, where the radius r may be any positive real number. It is an n-dimensional manifold in Euclidean (n + 1)-space.

The n-hypersphere (often simply called the n-sphere) is a generalization of the circle (called by geometers the 2-sphere) and usual sphere (called by geometers the 3-sphere) to dimensions n>=4. The n-sphere is therefore defined (again, to a geometer; see below) as the set of n-tuples of points ($x_1, x_2, ..., x_n$) such that

$$x_1^2 + x_2^2 + ... + x_n^2 = R^2 \quad (1)$$

where R is the radius of the hypersphere.

Let V_n denote the content of an n-hypersphere (in the geometer's sense) of radius R is given by

$$V_n = \int_0^R S_n r^{n-1} dr = \frac{S_n \cdot R^n}{n}$$ where S_n is the hyper-surface area of an n-sphere of unit radius. A unit hypersphere must satisfy

$$S_n \int_0^\infty e^{-r^2} r^{n-1} dr = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} .... \int_{-\infty}^{\infty} e^{-(x_1^2+...+x_n^2)} dx_1 .... dx_n = (\int_{-\infty}^{\infty} e^{-x^2} dx)^n \Rightarrow \frac{1}{2} S_n \Gamma(n/2) = (\Gamma(1/2))^n$$

And to the end

$$S_n = R^{n-1} 2(\Gamma(1/2))^n / \Gamma(n/2) = R^{n-1}(2\pi^{n/2})/\Gamma(n/2) \quad (1)$$

$$V_n = R^n (\pi^{n/2})/\Gamma(1+n/2) \quad (2)$$ But the gamma function can be defined by $\Gamma(m) = 2\int_0^\infty e^{-r^2} r^{2m-1} dr$

Strangely enough, the hyper-surface area reaches a maximum and then decreases towards 0 as n increases. The point of maximal hyper-surface area satisfies

$$\frac{dS_n}{dn} = R^{n-1}(2\pi^{n/2})/\Gamma(n/2) = R^{n-1}\pi^{n/2} \cdot [\ln \pi - \psi_0(n/2)]/\Gamma(n/2) = 0 \quad (3)$$

Where $\psi_0(x) = \Psi(x)$ is the digamma function.

For maximum volume the same they be calculated

$$\frac{dV_n}{dn} = R^n (\pi^{n/2})/\Gamma(1+n/2) = R^n \pi^{n/2} \cdot [\ln \pi - \psi_0(1+n/2)]/(2 \cdot \Gamma(1+n/2)) = 0 \quad (4)$$

From Feng Qi and Bai −Ni-Guo exist theorem

For all $x \in (0,\infty)$, $\ln(x+\frac{1}{2}) - \frac{1}{x} < \psi(x) < \ln(x+e^{-\gamma}) - \frac{1}{x}$ the constant $e^{-\gamma} = 0.56$.
Taking advantage of the previous theorem solved in two levels ie...

From (3) we have 2 levels :

$$\ln(\frac{1}{2}x + \frac{1}{2}) - \frac{1}{\frac{1}{2}x} = \ln \pi \quad (a) \quad \text{and} \quad \ln(\frac{1}{2}x + e^{-\gamma}) - \frac{1}{\frac{1}{2}x} = \ln \pi \quad (b)$$

Both cases, if resolved in accordance with the theorem (G.R.E) from by the form..

$$z = 2 \cdot (e^{\zeta} - 1/2) + \sum_{i=1}^{\infty} (\frac{(-m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (2 \cdot e^{\zeta} \cdot (\frac{-2}{2(e^{\zeta} - 1/2)})^{\zeta}))$$

with $\zeta \to \log(\pi)$ but $m = 1/e^{s+1}$, with s>=1 as before in 1 case. The initial value for (a) case is 5.59464 and for (b) case is 5.48125. We use the method approximate of Newton after obtaining an initial root z_s with s =1 is 7.27218 and 7.18109 respectively, finally after a few iterations.This shows that ultimately we as integer result the integer 7, for maximum hyper-surface area.

Thereafter for the case of maximum volume, and before applying From Feng Qi and Bai –Ni-Guo

For all $x \in (0, \infty)$, $\ln((\frac{1}{2}x + 1 + \frac{1}{2}) - \frac{1}{(\frac{1}{2}x+1)} = \log(\pi)$ and $\ln(\frac{1}{2}x + 1 + e^{-\gamma}) - \frac{1}{\frac{1}{2}x+1} = \ln \pi$. The

results in both cases according to equation..
$$z = 2 \cdot (e^{\zeta} - 3/2) + \sum_{i=1}^{\infty} (\frac{(-m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (2 \cdot e^{\zeta} \cdot (\frac{-2}{2(e^{\zeta} - 3/2) + 2})^{\zeta}))$$ with $\zeta \to \log(\pi)$ but $m = 1/e^{s+1}$ with s>=1 as before case. In two cases end up in the initial values 3.59464 and 3.48125. We use the method approximate of Newton arrive quickly in 5.27218 and 5.18109 respectively. Therefore the integer for the maximum volume hyper-surface is the 5.

3. THE KEPLER'S EQUATION

The kepler's equation allows determine the relation of the time angular displacement within an orbit. Kepler's equation is of fundamental importance in celestial mechanics, but cannot be directly inverted in terms of simple functions in order to determine where the planet will be at a given time. Let M be the mean anomaly(a parameterization of time) and E the eccentric anomaly (a parameterization of polar angle) of a body orbiting on an ellipse with eccentricity e, then ...

$$j = \frac{1}{2} a \cdot b \cdot (E - e \cdot SinE) \Rightarrow M = E - e \cdot SinE = (t - T) \cdot \sqrt{\frac{a^3}{\mu}} \text{ and } h = \sqrt{p \cdot \mu}$$

is angular momentum, $j = Area - angular$.Eventually the equation of interest is in final form is $M = E - e \cdot SinE$ and calculate the E. The Kepler's equation has a unique solution,but is a simple transcendental equation and so cannot be inverted and solved directly for E given an arbitrary M. However, many algorithms have been derived for solving the equation as a result of its importance in celestial mechanics. In essentially trying to solve the general equation $x - e \cdot Sinx = t$ where t, e are arbitrary in C more generally. According to the theory G.R.E we have two

basic cases $P_1(z) = z = \zeta$ (a) and $P_2(z) = Sin(z) = \zeta$ (b) which if the solve separately,the total settlement will result from the union of the 2 fields of the individual solutions.The first case is this is of interest in relation to the equation Kepler, because $e < 1$.From theory G.R.E we have the solution

$$z = \zeta + \sum_{i=1}^{\infty} (\frac{(e)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (\zeta' . Sin^i(\zeta))) = \zeta + \sum_{i=1}^{\infty} (\frac{(e)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (Sin^i(\zeta))) \ (1)$$

for $\zeta \rightarrow t$. Since the exponents are changed from an odd to even we use two general expressions for the nth derivatives. If we have even exponent is

$$\frac{\partial^{2n-1}}{\partial x^{2n-1}} Sin^{2n}(x) = (1/2^{2*n-1}) * \sum_{k=0}^{n-1}(-1)^{n-k} * (2*n)!/(k! * (2n-k)!) * (2n - 2k)^{2n-1} * Sin[(2n-2k)*t + (2n)\pi/2]$$

and for odd exponent is

$$\frac{\partial^{2n}}{\partial x^{2n}} Sin^{2n+1}(x) = \frac{1}{2^{2n}} * \sum_{k=0}^{n}(-1)^{n-k} * (2*n+1)!/(k! * (2n+1-k)!) * (2n-2k+1)^{2n} * Sin[(2n-2k+1)*x + (2n)\pi/2]$$

These formulas help greatly in finding the general solution of equation Kepler,because this is generalize the nth derivative of $Sin^i(\zeta)$ as sum of the two separate cases. So from (1) we can see the only solution for the equation Kepler's with the type (2)

$$z = t + \sum_{n=0}^{\infty} (1/2^{2n}) * \sum_{k=0}^{n}(-1)^{n-k} * ((m)^{2*n+1}/Gamma[2*n+2]) * (2*n+1)!/(k! * (2n+1-k)!)$$
$$* (2n-2k+1)^{2n} * Sin[(2n-2k+1)*t + (2n)\pi/2]$$
$$+ \sum_{s=0}^{\infty} (1/2^{2*s-1}) * \sum_{k=0}^{s-1}((m)^{2*s}/Gamma[2*s+1]) * (-1)^{s-k} * (2*s)!/(k! * (2s-k)!)$$
$$* (2s-2k)^{2s-1} * Sin[(2s-2k)*t + (2s)\pi/2]$$

The second case solution of the $x - e \cdot Sinx = t$ according to the theory G.R.E we can also from the $P_2(z) = Sin(z) = \zeta$ (b) that $z = ArcSin(z) + 2k\pi$ and also $z = -ArcSin(z) + (2k+1)\pi$. So the full solution of the equation $x - e \cdot Sinx = t$ of the second field of roots is ...

$$z_k = (ArcSin(\zeta) + 2k\pi) + \sum_{i=1}^{\infty} (\frac{(1/e)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} ((ArcSin(\zeta)'\cdot(ArcSin(\zeta)+2k\pi)^i)) \quad (3)$$

Or also

$$z_k = (-ArcSin(\zeta) + (2k+1)\pi) + \sum_{i=1}^{\infty} (\frac{(1/e)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} ((-ArcSin(\zeta)'\cdot(-ArcSin(\zeta)+(2k+1)\pi)^i)) \quad (4)$$

$$ArcSin(\zeta)' = \frac{1}{\sqrt{1-\zeta^2}} \quad (5)$$

with $\zeta \to t/e$ and $k \in Z$. An example is the Jupiter, with data $M = 5 \cdot 2 \cdot \pi / 11.8622$ with eccentricity e where $e = 0.04844$, then from equation (2) we find the value of (x or E) = 2.6704 radians.

4. THE NEUTRAL DIFFERENTIAL EQUATIONS (D.D.E)

In this part solve of transcendental equations we introduce another class of equations depending on past and present values but that involve derivatives with delays as well as function itself. Such equations historically have been referred as neutral differential difference equations.

The model non homogeneous equation is

$$\sum_{k=1}^{n} g_k \cdot \frac{\partial^k}{\partial x^k} x(t) + \sum_{r=1}^{m} c_r \cdot \frac{\partial^r}{\partial x^r} x(t-\tau_r) = a \cdot x(t) + \sum_{i=1}^{\sigma} w_i \cdot x(t-v_i) + f(t) \quad (1)$$

With g_k, c_r, a, w_i is constants and $w_i \neq 0$ and $f(t)$ is a continuous function on C. Of course any discussion of specific properties of the characteristic equation will be much more difficult since this equation transcendental, will be of the form :

$$h(\lambda) = a_0(\lambda) + \sum_{j=1}^{n1} a_j(\lambda) \cdot e^{-\lambda \tau_j} + \sum_{i=1}^{n2} b_i(\lambda) \cdot e^{-\lambda v_i} = 0 \quad (2)$$

Where $a_j(\lambda), b_i(\lambda), j > 0$ are polynomials of degree $\leq (m+\sigma)$ and $a_0(\lambda)$ is a polynomial of degree n also must $n_1 + n_2 \leq m + \sigma$. The equations (2) also resolved in accordance with the method G.R.E and the general solution is of as the form $x(t) = f_s(t) + \sum_j p_j(t) \cdot e^{\lambda_j \cdot t}$ where λ_j are the roots of the equation of characteristic and p_j are polynomials and also $f_s \neq f$ in generally. As an example we give the D.D.E differential equation $x'(t) - C \cdot x'(t-r) = a \cdot x(t) + w \cdot x(t-v) + f(t)$ (3) which is like an equation $h(\lambda)$ as of characteristic $h(\lambda) = \lambda(1 - C \cdot e^{-\lambda \cdot r}) - a - w \cdot e^{-\lambda \cdot v} = 0$ where $C \neq 0, r \geq 0, v \geq 0$ and a, w constants.

1. SOLUTION OF THE EQUATION $x^x - m \cdot x + t = 0$

The solution of the equation is based mostly on the solution of equation $x^x = z$ which has solution relying on the solution of $x \cdot e^x = v$ which solved before. Specifically because we know the function

$W_k(z)$ is product log function $k \in Z$, and using it to solve the equation $x \cdot e^x = v$ is $z = W_k(v), v \neq 0$.

also $k \in Z$, all the solutions of the equation $x^x = z$ is for $z \neq 0$, . According to this assumption we can solve the equation $x^x - m \cdot x + t = 0$ with the help of the method G.R.E. According to the theory G.R.E we have two basic cases $p_1(x) = x^x = \zeta$ **(a)** and $p_2(x) = x = \zeta$ **(b)** which if the solve separately, the total settlement will result from the union of the 2 fields of the individual solutions, $\zeta \in C$. The first case is of interest in relation to the equation has more options than the second, because it covers a large part of the real and the complex solutions. This situation leads to the solution for x such that it is in the form $x = e^{ProductLog[(2k)\pi i + Log[\zeta]]}$ or taking and the other form

$$x = \frac{(\log(\zeta))}{(W_k(\log(\zeta)))}$$

From theory G.R.E we have the solution

$$x_k = e^{ProductLog[h,2\pi ik + \log[\zeta]]} + \sum_{v=1}^{\infty} (\frac{(-m)^v}{Gamma(i+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} ((e^{ProductLog[h,2\pi ik + \log[\zeta]]})' \cdot e^{v \cdot ProductLog[h,2\pi ik + \log[\zeta]]})$$ with the $k \in Z$

and $h = -1, 0, 1$ or more exactly

$$x_k = e^{ProductLog[h,2\pi ik + \log[\zeta]]} + \sum_{v=1}^{\infty} (\frac{(-m)^v}{Gamma(v+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} (\frac{1}{\zeta \cdot (1 + ProductLog[h,2\pi ik + \log[\zeta]])}) \cdot e^{v \cdot ProductLog[h,2\pi ik + \log[\zeta]]})$$ with

multiple roots in relation to k and $\zeta \to t$. Variations presented in case where, when we change the sign of m, t mainly in the sign of the complex roots. Even and in anomaly in the approach of the infinite sum we use the transformation but $m_* = m / e^{s+1}$ with s>=1, a very good approximation also in this special case is when we use the method approximate of Newton after obtaining an initial root z_s. The second group of solutions represents real mainly roots of equation where $p_2(x) = x = \zeta$

So we have

$$x = \zeta + \sum_{v=1}^{\infty} (\frac{(-1/m)^v}{Gamma(v+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} (\zeta' \cdot \zeta^{v\zeta}) = \zeta + \sum_{v=1}^{\infty} (\frac{(-1/m)^v}{Gamma(v+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} (\zeta^{v\zeta})$$ with $\zeta \to t/m$, for $m, t \in C$

in generally.

1. **SOLUTION OF THE EQUATION** $x^q - m \cdot x^p + t = 0$

An equation seems simple but needs analysis primarily on the distinction of m, but also the powers specific p,q as to what look every time.

<u>Distinguish two main cases:</u>

i)   **p, q** $\in \mathbf{R}$

The weight method would follow it takes m, which regulates the method we will follow any time. But according to the method G.E.R we have two basic cases $P_1(x) = x^p = \zeta$ **(a)** and $P_2(x) = x^q = \zeta$ **(b)** whose solution gives the individual a comprehensive solution of the equation. For the case under consideration ie $m > 1, p > q$ transforms the original in two formats to assist us in connection with the logic employed by the general relation G.R.E.

The first transform given from the form $x^p - m \cdot x^q + t = 0 \Rightarrow x^q - (1/m) \cdot x^p - t/m = 0$ which is now in the normal form to solve equation. First we need to solve the relationship $x^p = \zeta$ in C. Following that we can get the form $x_k = e^{(Log(\zeta) + 2 \cdot k \cdot \pi \cdot i)/q}$, $k \in Z, k = 0, \pm 1, \pm 2, \ldots \pm IntegerPart[q/2]$ and the count of roots is maximum 2*IntegerPart[q/2] in generality.

Therefore so the first form of solution of the equation is..

$$x_k = e^{(Log(\zeta) + 2 \cdot k \cdot \pi \cdot i)/q} + \sum_{v=1}^{\infty} (\frac{(-1/m)^v}{Gamma(i+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} ((\frac{e^{(Log(\zeta) + 2 \cdot k \cdot \pi \cdot i)/q}}{q \zeta}) \cdot (e^{p \cdot (Log(\zeta) + 2 \cdot k \cdot \pi \cdot i)/q})^v)$$

Where $\partial_\zeta (e^{(Log(\zeta) + 2 \cdot k \cdot \pi \cdot i)/q}) = (e^{(Log(\zeta) + 2 \cdot k \cdot \pi \cdot i)/q})/(q \cdot \zeta)$, with multiple roots in relation to k and $k \in Z, k = 0, \pm 1, \pm 2, \ldots IntegerPart[q/2]$ and $\zeta \to t/m$.

But for the complete solution of this case and find the other roots of the equation for this purpose i make the transformation $x = y^{-1}$ and we have $x^p - m \cdot x^q + t = 0 \Rightarrow y^{-q} - m \cdot y^{-p} + t = 0$ and then we transform in $1 - m \cdot y^{p-q} + t \cdot y^p = 0 \Rightarrow y^{p-q} - t/m \cdot y^p - 1/m = 0$. In this way we find a whole other roots we have left from all the roots. The form of solution will be as above and assuming the that $g = p - q$ we have..

$$y_k = e^{(Log(\zeta) + 2 \cdot k \cdot \pi \cdot i)/g} + \sum_{v=1}^{\infty} (\frac{(t/m)^v}{Gamma(i+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} ((\frac{e^{(Log(\zeta) + 2 \cdot k \cdot \pi \cdot i)/g}}{g \zeta}) \cdot (e^{p \cdot (Log(\zeta) + 2 \cdot k \cdot \pi \cdot i)/g})^v)$$ and $x_k = 1/y_k$ which roots

are in relation to $k \in Z, k = 0, \pm 1, \pm 2, \ldots \pm IntegerPart[g/2]$ with $\zeta \to -1/m$. The second case related to m<1 has no procedure for dealing with the method. Starting from the original equation was originally

found on the p and so the first transform given from the form $x^p - m \cdot x^q + t = 0$ to solve the relationship $x^p = \zeta$ in C, as helpful to the general equation G.R.E. So we have

$$x_k = e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/p} + \sum_{v=1}^{\infty} \left(\frac{(-m)^v}{Gamma(i+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} \left(\left(\frac{e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/p}}{p \cdot \zeta} \right) \cdot (e^{q \cdot (Log(\zeta)+2 \cdot k \pi \cdot i)/p})^v \right) \right)$$

with $k \in Z, k = 0, \pm 1, \pm 2, \ldots \pm IntegerPart[p/2]$ with $\zeta \to t$.

To settle the issue of finding the roots, where roots arise other and with m<1 then i make the transformation $x = y^{-1}$ and we have $x^p - m \cdot x^q + t = 0 \Rightarrow y^{-q} - m \cdot y^{-p} + t = 0$ and then we transform in $y^q + 1/t \cdot y^{q-p} - m/t = 0$ with the pre case $p < q$. This transformation is relevant to the case remains as a final case before us. The solution in this case has form and assuming the that $g = q - p$ we have..

$$y_k = e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/q} + \sum_{v=1}^{\infty} \left(\frac{(-1/t)^v}{Gamma(i+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} \left(\left(\frac{e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/q}}{q \cdot \zeta} \right) \cdot (e^{g \cdot (Log(\zeta)+2 \cdot k \cdot \pi i)/q})^v \right) \right)$$ and $x_k = 1/y_k$ which roots

are in relation to $k \in Z, k = 0, \pm 1, \pm 2, \ldots \pm IntegerPart[q/2]$ with $\zeta \to m/t$.

i) $\mathbf{p, q \in C}$

In this case should first solve the equation ,$z^q - m \cdot z^p + t = 0, z \in C$.The solution for z variable, after several operations in concordance with the type De Moivre, we get the relation connecting

the real and imaginary parts the general case of complex numbers..$z^{a+bi} = x + yi$

and the solution is

$$z_k = e^{\frac{b(2k\pi+Arg(x+yi))}{a^2+b^2}} (x^2 + y^2)^{\frac{a}{2(a^2+b^2)}} Cos[\frac{a(2k\pi + Arg(x+yi)}{a^2+b^2} - \frac{bLog[x^2+y^2]}{2(a^2+b^2)}] +$$

$$ie^{\frac{b(2k\pi+Arg(x+yi))}{a^2+b^2}} (x^2 + y^2)^{\frac{a}{2(a^2+b^2)}} Sin[\frac{a(2k\pi + Arg(x+yi)}{a^2+b^2} - \frac{bLog[x^2+y^2]}{2(a^2+b^2)}]$$

we see that the number of solutions, resulting from the denominator of the fraction that the full line equals with the $c = (a^2 + b^2)/a$ if prices of $k \in Z, k = 0, \pm 1, \pm 2, \ldots \pm IntegerPart[c/2]$. For the case under consideration ie $m > 1, p > q$ transforms the original in two formats to assist us in connection with the logic employed by the general relation G.R.E.

The first transform given from the form $x^p - m \cdot x^q + t = 0 \Rightarrow x^q - (1/m) \cdot x^p - t/m = 0$ which is now in the normal form to solve equation. First we need to solve the relationship $x^p = \zeta$ in C. Following that we can get the form $k \in Z, k = 0, \pm 1, \pm 2, \ldots \pm IntegerPart[q/2]$ and the count of roots is maximum 2*IntegerPart[q/2] in generality.The solution is when we analyze the power as

$$y_k = e^{(Log(\zeta)+2\cdot k\cdot\pi\cdot i)/q} + \sum_{v=1}^{\infty} \left(\frac{(-1/m)^v}{Gamma(v+1)} \frac{\partial^{v-1}}{\partial\zeta^{v-1}} \left(\left(\frac{e^{(Log(\zeta)+2\cdot k\cdot\pi\cdot i)/q}}{q\zeta}\right)\cdot(e^{p\cdot(Log(\zeta)+2\cdot k\cdot\pi\cdot i)/q})^v\right)\right)$$ and $x_k = y_k$ which roots are

in relation to $k \in Z, k = 0, \pm1, \pm2, \ldots \pm IntegerPart[q/2]$ with $\zeta \to t/m$. **The remaining cases are similar to**

previous with $\mathbf{p, q} \in \mathbf{R}$. **The sole change is in relation to the number of cases is**

$Integer((a^2+b^2)/a)$ **for (+/- x axes) and** $z^{a+bi} = x + y\cdot i = w$ **for any** $w, z \in C$.

1. 2 FAMOUS EQUATIONS OF PHYSICS

i)The difraction phenomena due to "capacity" of the waves bypass obstacles in their way, so to be observed in regions of space behind the barriers, which could be described as geometric shadow areas .

In essence the phenomena of diffraction phenomena is contribution, that is due to superposition of waves of the same frequency that coexist at the same point in space.

If virus is where the intensity at a distance ro from the slot at $\theta = 0$, ie opposite to the slit. So finally we write the relationship in the form

$$I(\theta) = I_o \frac{\sin^2 w}{w^2}$$

$$w = \frac{1}{2}kD\sin\theta$$

The maximum intensity appears to correspond to the extremefunction sin w / w.Derivative of and equating to zero will take the trigonometric equation w = **tan** w a solution which provides the values of w corresponding to maximum intensity. With the assist of a second of the relations We can then, for a given problem is know the wave number k (or wavelength λ) and width D the slit, to calculate the addresses corresponding to θ are the greatest.

Consider many tears as a crowd of 2 N +1 parallel between the cracks width D, the distance from center to center is a and which we have numbered from – N to N. Such a device called a diffraction grating slits.We accept that sufficiently met the criterion for Fraunhofer diffraction and find the equation for the volume.

$$I(\theta) = I_o \frac{\sin^2 w}{w^2} \frac{\sin^2 Mu}{\sin^2 u}$$

where

$$w = \frac{\pi D \sin\theta}{\lambda}$$

$$u = \frac{\pi a \sin\theta}{\lambda}$$

There fringes addresses for which zero quantity sin u, and therefore the intensity of which is determined by the factor

$$\sin^2 w / w^2$$

So we must solve the relation w = **tan** w.

Where $k=\alpha/D$ and $u=kw$, $m=M$. Trying solving the general form of the equation w = **m*tan** w with $m \in C$, consider 2 general forms of solution, arising from the form $Cos(w)=\zeta$ and $Cos(w) = \zeta \Rightarrow w = \pm ArcCos(w) + 2k\pi$

$k \in Z, k = 0,\pm 1,\pm 2,...$ **so** we have..

$$w_p = (ArcCos(\zeta)+2k\pi)+\sum_{i=1}^{\infty}(\frac{(m)^i}{Gamma(i+1)}\frac{\partial^{i-1}}{\partial\zeta^{i-1}}((ArcCos(\zeta))\cdot(Sin[ArcCos(\zeta)+2k\pi)]/(ArcCos(\zeta)+2\kappa\pi))^i)$$

and the form

$$w_q = (-ArcCos(\zeta)+2k\pi)+\sum_{i=1}^{\infty}(\frac{(m)^i}{Gamma(i+1)}\frac{\partial^{i-1}}{\partial\zeta^{i-1}}((-ArcCos(\zeta))\cdot(Sin[-ArcCos(\zeta)+2k\pi)]/(-ArcCos(\zeta)+2\kappa\pi))^i$$

Then the general solution is $w_q \cup w_p$.

ii) The spectral density of black body is given by the equation

$$u(\nu) = \bar{E}\rho(\nu) = \frac{h\nu}{e^{bh\nu}-1}\frac{8\pi\nu^2}{c^3} = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1}$$

according to the relationship of Plank.

The correlated u (λ)

$$u(\lambda) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/kT\lambda}-1}$$

By $c = \lambda/T = \lambda\nu$ which is extreme if the derivative zero. Thus we have the relationship

$$\frac{d}{d\lambda}u(\lambda) = 8\pi hc\frac{-5\lambda^4\left(e^{hc/kT\lambda}-1\right)-\lambda^5 e^{hc/kT\lambda}\left(-\frac{1}{\lambda^2}\frac{hc}{kT}\right)}{\lambda^{10}\left(e^{hc/kT\lambda}-1\right)^2}$$

Zeroing the derivative will have the relationship

$$-5\left(e^{hc/kT\lambda}-1\right)+e^{hc/kT\lambda}\left(\frac{1}{\lambda}\frac{hc}{kT}\right) = 0$$

And if $x = hc/kT\lambda$ then we get the equation

$$5 - 5e^{-x} - x = 0$$

Finding the solution of x we find the relationship

$$\lambda_{max}T = b$$

$$5 - 5e^{-x} - x = 0$$

Finding the solution of x we find the relationship

$$\lambda_{max} T = b$$

By $b = hc / 4.965 \cdot k$ Is called constant Bin, called displacement law. Then we need to calculate the general solution of the equation by the method G,R.E.

The first group of solutions represents real mainly roots of equation where $p_1(x) = x = \zeta$

So we have

$$x = \zeta + \sum_{v=1}^{\infty} (\frac{(m)^v}{Gamma(v+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} (\zeta' \cdot Exp[-\zeta])^v) = \zeta + \sum_{v=1}^{\infty} (\frac{(m)^v}{Gamma(v+1)} (-v^{v-1} e^{-\zeta v}) \quad \text{with } \zeta \to t \, ,$$

for $m,t \in C$. In this case t=5 and m=-5, we calculate the x=4.9651142317442763037 the nearest 20 ignored. Because apply relation

$$\frac{\partial^r}{\partial x^r} e^{-xw} = (-w)^r e^{-xw}$$

$$\frac{\partial^r}{\partial x^r} e^{xw} = (w)^r e^{xw}$$

The second group of solutions represents complex roots of equation where
$p_2(x) = e^x = \zeta \Rightarrow x = \log(\zeta) + 2\kappa\pi i \cdot$

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